The Dirac Equation and the Superluminal Electron Model

By Richard Gauthier, Ph.D.

The Dirac equation (1928) is one of the most successful equations of quantum mechanics, the theory of matter and energy at atomic and sub-atomic levels. Physicist and historian of physics Abraham Pais (1994) has ranked the Dirac equation "among the highest achievements of twentieth-century science."

When quantum mechanics was being established in the mid 1920's, Paul A. M Dirac did not seek a visualizable model of the electron or other quantum mechanical objects. But a number of the quantitative physical properties of the electron described by the Dirac equation are of a spatial or temporal nature that can lend themselves to visualizable physical modeling. A recently proposed superluminal quantum model of the electron by the author (Gauthier, 2005) has several of these characteristics. The present article summarizes some of the main quantitative attributes of the Dirac electron and describes how the superluminal model of the electron embodies a number of these attributes. The utility of any physical model is to help deepen one's understanding of a physical process or object corresponding to the model, and to suggest new experiments. Whether the superluminal model of the electron can serve this purpose for the Dirac electron is left for the reader to decide.

Brief historical summary of quantum ideas up to Dirac

In 1900, Max Planck mathematically explained the distribution of frequencies of radiation from hot objects called "black bodies". He proposed that radiant energy could be emitted and absorbed from small oscillators in these bodies only in certain discrete amounts he called quanta. The energy radiated was proportional to the frequency of the oscillators. The proportionality constant later became known as Planck's constant. This was the beginning of quantum theory.

In 1905 Albert Einstein explained the emission of electrons from the surface of a metal when light above a certain frequency is shined on it (the photoelectric effect). He proposed that light itself was composed of particles, where the energy in each light particle (later called a photon) was proportional (using Planck's constant) to its frequency.

In 1911, based on experimental evidence of sub-atomic alpha particles scattered from thin gold foil, Ernest Rutherford proposed a model of the atom having a very small, heavy positively charged nucleus surrounded by negatively charged electrons. But this model was inadequate because according to the known laws of electromagnetism, the electrons would have immediately radiated away their energy and spiraled into the nucleus.

In 1913, Niels Bohr modeled the atom as resembling a miniature solar system, with negative electrons circling a small, positively charged nucleus only in certain allowed stationary orbits and energy levels. Electrons could radiate or absorb light energy of a particular frequency (based on Planck's constant) when jumping from one stationary orbit to another. Bohr's planetary model of the atom was a hybrid of pre-quantum physics and early quantum ideas, along with some rules that Bohr supplied to make his model fit some of the known facts about atoms. The model yielded the orderly spectrum of wavelengths of light that can come from hydrogen atoms. These wavelengths had earlier been described by an empirically derived mathematical formula. Although the Bohr model worked rather well in accounting for the light spectrum of hydrogen and a few other atoms and charged ions having one optically active electron, it was unable to account for the spectrum of frequencies coming from atoms like helium having two or more optically active electrons.

In a more refined Bohr planetary model of the atom, three quantum numbers were needed to describe the orbit of an electron in an atom. A fourth quantum number was introduced by Wolfgang Pauli in 1924 in order to describe a quantum mechanical "two-valuedness" which caused an observed splitting of certain spectral frequencies. It was this fourth quantum number that was later associated with the electron's spin by George Uhlenbeck and Samuel Goudsmit in 1925. No acceptable visual model for the spinning electron was ever provided. Today the electron is considered to be point-like and not visualizable.

Also in 1925, Werner Heisenberg and his colleagues developed a more accurate quantum mechanics called matrix mechanics. This approach was based on measured light frequencies and intensities coming from atoms when electrons transition from one energy level to another in an atom. With this abstract mathematical formalism Heisenberg renounced the idea of visual models of the atom or sub-atomic particles. In 1927 he proposed his indeterminacy relation, often called the Heisenberg uncertainty principle, that came from matrix mechanics. This relationship sets a lower limit, related to Planck's constant, on the accuracy with which certain pairs of physical quantities of a particle, such as its position and momentum, can be measured together.

In 1926, Erwin Schrödinger introduced his highly successful wave mechanics, based partly on the 1924 matter-wave description of the electron by Louis de Broglie. Schrödinger hoped that his wave equations for the electron would provide a picture for the structure of the charge distribution of an electron. But his hopes were not realized. The wave equations were shown to be useful only for predicting the probability that point-like electrons would be found in the regions of space where the waves were present. Schrödinger and Dirac independently proved that the Heisenberg and Schrödinger approaches to quantum mechanics are mathematically equivalent and therefore make the same statistical predictions about atomic events.

In 1927 Pauli mathematically incorporated the idea of electron spin into the quantum mechanics of Heisenberg and Schrödinger. Though this revised quantum mechanics was very useful and accurate for some atoms and molecules, it was not consistent with Einstein's theory of special relativity, and so was inaccurate when electron speeds in atoms approached the speed of light. Schrödinger and others had tried without success to get a relativistic wave equation for the electron.

Finally in 1928 Dirac introduced his relativistic wave equation. He had wanted to find an equation for the electron that would be consistent with special relativity and describe the known fine structure frequency spectrum of hydrogen. In technical terms the equation was Lorentz-covariant. It correctly described the fine structure of hydrogen. When Dirac examined the solutions to his equation, he found that, much to his surprise, it also predicted the electron's spin and its magnetic property called the magnetic moment. Perhaps even more surprising, his equation also suggested a positively charged particle with the mass of the electron could exist. Based on his equation, in 1931 Dirac predicted what he called the anti-electron (now called a positron). It was confirmed experimentally in 1932. For his work in quantum mechanics Dirac shared with Schrödinger the Nobel Prize for physics in 1933.

Dirac's bias against forming visual models in quantum mechanics

Despite the success of Heisenberg's and Schrödinger quantum mechanics, Dirac's relativistic equation, and the later quantum electrodynamics theory in predicting the statistical results of sub-atomic, atomic and molecular experiments, none of these physical theories gives a visualizable image of what is actually going on behind the symbolism of the equations, and in particular none provides a visual model for the structure of the electron.

In developing his equation, Dirac assumed that the electron was a point-like particle with an electric charge equal to the experimentally measured charge of the electron. Dirac however did not try to develop visualizable physical models as aids to picturing or understanding his mathematical results. "Beware of forming models or mental pictures at all," Dirac once said to Schrödinger (Cline, 1965) At another time Dirac summarized this view about visualizability that was also strongly advocated by Heisenberg: "The main object of physical science is not the provision of pictures, but is the formulation of laws governing phenomena and the applications of these laws to the discovery of new phenomena. If a picture exists, so much the better; but whether a picture exists or not is a matter of only secondary importance." (Cropper, 2001)

Dirac's and Heisenberg's attitude of not seeking and even discouraging visualizable models underlying quantum phenomena came to dominate quantum theorizing. Albert Einstein took quite a different approach to visual modeling: "If I can't picture it, I can't understand it." (Wheeler, 1991)

Arthur Eddington, the astrophysicist who led the solar eclipse expedition in 1919 that confirmed Einstein's prediction about the bending of starlight as it passes the sun, lamented (Eddington, 1928) the lack of a picture of the electron: "…we have perhaps forgotten that there was a time when we wanted to be told what an electron is. The question was never answered. No familiar conceptions can be woven round the electron, it belongs to the waiting list."

More recently, physicist A.O. Barut (1991) summarized the problem of visualizing the structure of the electron: "If a spinning particle is not quite a point particle, nor a solid three dimensional top, what can it be? What is the structure which can appear under probing with electromagnetic fields as a point charge, yet as far as spin and wave properties are concerned exhibits a size of the order of the Compton wave length?" The Compton wavelength is the wavelength of a photon having the same energy as is contained in the mass of a resting electron, based on Einstein's equation $E = mc^2$.

The remarkable successes of the Dirac equation

One highly condensed form of the Dirac equation for a free electron (inscribed on a plaque in Westminster Abbey) is given by

$$i\gamma \cdot \partial\psi = m\psi$$

where $i = \sqrt{-1}$, γ represents four 4-by-4 square arrays of numbers, ∂ symbolizes rate-of-change calculations with respect to the 3 dimensions of space and one of time, ψ is a four-component wave equation for the electron, and m is the rest mass of the electron.. A more expanded version of the Dirac equation for a free electron, with slightly different symbolism

$$\left[\left(-\frac{\hbar}{ic}\frac{\partial}{\partial t}\right) - \sum_{r=1}^{3}\alpha_r\left(\frac{\hbar}{i}\frac{\partial}{\partial x_r}\right) - \alpha_0 mc\right]\psi\,(\mathbf{x},t) = 0$$

where $\alpha_0, \alpha_1, \alpha_2$ and α_3 are 4-by-4 arrays of numbers, $\hbar$ is Planck's constant h divided by 2π, c is the speed of light, and m is the rest mass of the electron.

The Dirac equation had four principal successes:

1) It predicted the known spin or angular momentum $\frac{1}{2}\hbar$ of the electron.

2) It was the first electron equation in quantum mechanics to satisfy Lorentz-covariance, an important restriction on physical theories from Einstein's special theory of relativity. The Lorentz-covariant Dirac equation precisely yielded the hydrogen atom's known fine structure frequency spectrum of light and provided the four quantum numbers that describe each electron in an atom.

3) It predicted the known magnetic property of the electron, called the magnetic moment of value $e\hbar/2m$.

4) It led to Dirac's prediction of previously unknown antimatter -- the anti-electron or positron. That prediction was confirmed when the positron, the anti-particle of the electron, having equal mass but opposite charge as the electron, was experimentally detected by Carl Anderson in 1932. The concept of antimatter and its relation to matter and to light led to the development of the modern theory of

quantum electrodynamics (QED), which also improved on some of the predictions of the Dirac equation.

Problematical predictions of the Dirac equation

There are several results of Dirac's equation which were problematical, despite the equation's general success:

1) Dirac's equation seemed to predict that electron kinetic energies could be negative. He interpreted this result as due to the existence of an infinite sea of negative energy electrons that is physically undetectable. When an electron gains enough energy and jumps out of this electron sea, the hole left behind is positively charged. If a positive hole meets an electron they will annihilate each other, producing photons. Dirac's electron sea model raised concerns for some time. At first Dirac thought the positively charged hole was a proton, which is about 1800 times the mass of an electron, but that didn't work. The electron sea model was better accepted several years later, when Dirac interpreted the negative energy solution of his equation as being associated with a positron -- an anti-particle having the same mass as an electron but having a positive charge. The electron sea view of positrons was later revised by Richard Feynman, who, following an idea from John Wheeler, developed a quantum electrodynamics theory where a positron is an electron traveling backward in time. This is the currently accepted view, but according to Feynman it is "exactly equivalent to the Dirac negative energy sea point of view."(1965)

2) The electron instantaneously moves at the speed of light. This was a surprising result because the electron is always observed experimentally to move at less than the speed of light. Dirac explained that the observed sub-luminal speed of the electron is an average speed, not the instantaneous electron speed which is the speed of light. Another way to look at this is that the position of the electron's charge, which circulates at the velocity *c*, is not located at the center of mass of the electron, which moves at a velocity less than *c*.

3) An electron has an oscillatory motion with the extremely high frequency of $2f_0 = 2mc^2/h$ or 2.5×10^{20} cycles per second in addition to its normal linear motion. Schrödinger labeled that rapid oscillatory motion *Zitterbewegung* or "jittery motion".

4) This oscillatory motion of the electron was found by Schrödinger to have a minute amplitude of $\frac{1}{2}\hbar/mc$ or about 1.9×10^{-13} meters.

5) The electron has a mathematical rotation property that requires a 720-degree rotation to bring the electron back to its original state. A normal 3-D physical object returns to its original position after a 360-degree rotation. This

mathematical property of the electron has contributed to the difficulty of making a visualizable model of the electron.

6) Using the Dirac equation, de Broglie showed that for a free electron at rest or in uniform linear motion with no external forces acting on it, the “jittery motion” of the electric charge caused the linear momentum of the charge to not be exactly constant or conserved. It is only the electron’s average linear momentum that is conserved in the electron’s motion.

7) Besides predicting the magnetic moment, Dirac found that his equation also predicts an oscillating electric dipole moment of an electron, of magnitude $ie\hbar / 2mc$. The value i is the quantity $\sqrt{-1}$ which is called an imaginary number in mathematics but which has well-known and useful mathematical properties and also appears in the Dirac equation itself. But because of the appearance of the imaginary quantity i in the expression for the electric moment, Dirac wrote in his 1928 article introducing his equation that “The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning...” In his book “Principles of Quantum Mechanics” (1983) he says, referring to the dipole moment expression, that “these extra terms involve some new physical effects, but since they are not real they do not lend themselves very directly to physical interpretation.”

8) Like the electron’s mathematically imaginary electric moment, the electron’s intrinsic spin and its associated magnetic moment are considered to be paradoxical and problematical from a classical or pre-quantum standpoint. The electron’s spin is accepted because it provides a fourth quantum number for atomic electrons and helps explain known spectroscopic frequency data. But the spin is paradoxical because the electron seems point-like, with a size no larger than 10^{-18} meters. The electron’s magnetic moment is also accepted because its physical effects are detected experimentally. But it is also paradoxical because a magnetic moment would normally be created by a current moving in a finite-sized loop, or by a point charge moving in a finite-sized closed circulating path. However, the electron is experimentally found to be point-like. And no closed circulating path of a point-like charge that would produce the electron’s magnetic moment has ever been observed.

The following is a quote from Dirac's Nobel Prize acceptance speech (1933) referring to the electron’s jittery motion: "The variables α (alpha in his equation) also give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

The superluminal model of the electron

It was mentioned above that Dirac did not in this period attempt to make a physical model of the electron and did not support such attempts. However, many of the properties of the Dirac electron, including the problematical properties above, lend themselves to a physical modeling approach, which may help resolve some of these problematical properties

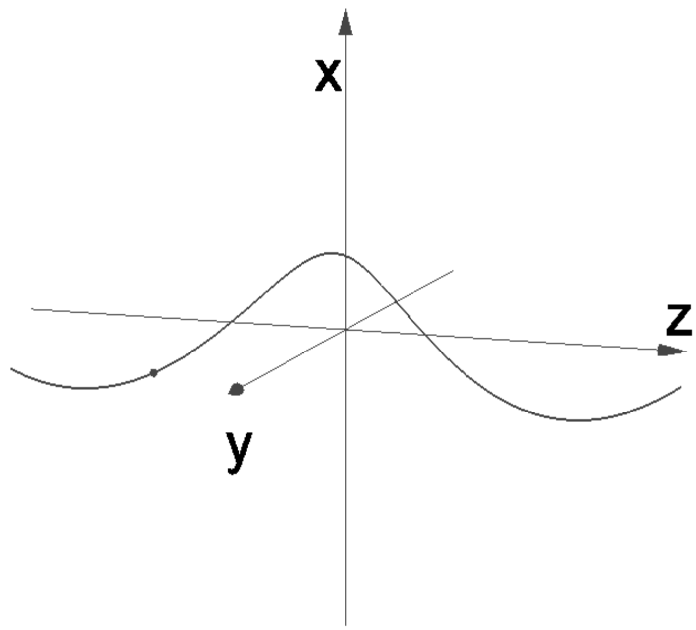
Recently a superluminal quantum model of the electron and the photon (Gauthier, 2005) was proposed. The electron model has a number of the characteristics of the Dirac electron as well as a visualizable internal quantum trajectory structure. The electron model is composed of a circulating superluminal photon-like object. The superluminal photon model first needs to be briefly described.

The superluminal model of the photon is appropriate for a photon of any wavelength λ. The model is composed of an uncharged point-like quantum that moves along an open helical trajectory having a radius R which depends on its wavelength λ. One turn of the helical trajectory corresponds to one wavelength λ of the photon model. The helical trajectory can turn clockwise or counterclockwise as the quantum moves forward, corresponding to a right or left circularly polarized photon. The radius R of the helical trajectory of the quantum is found to be $R = \lambda / 2\pi$. The forward angle of the helical trajectory is found to be 45 degrees. The speed of the superluminal photon along its helical trajectory is found to be $\sqrt{2}c = 1.414c$ where c is the speed of light. The superluminal quantum has a frequency f given by $f = c/\lambda$ or equivalently $\lambda = c/f$. The photon's angular frequency ω is given by $\omega = 2\pi f = 2\pi c/\lambda$. The quantum has an energy E given by $E = hf$ where h is Planck's constant. This is the experimental relationship for the energy of a photon. The quantum by its helical motion also has a spin or angular momentum of magnitude $s = h/2\pi = \hbar$ which is the experimental value of the spin of a photon.

The equations for the trajectory of the superluminal quantum moving forward and clockwise in the z direction (to the right in the image below) are given by

$$x = \frac{\lambda}{2\pi}\cos(\frac{2\pi}{\lambda}ct) = \frac{\lambda}{2\pi}\cos(\omega t)$$
$$y = \frac{\lambda}{2\pi}\sin(\frac{2\pi}{\lambda}ct) = \frac{\lambda}{2\pi}\sin(\omega t)$$
$$z = ct$$

The 3-D trajectory of this right-handed superluminal photon looks like this.



Superluminal photon model.
The uncharged superluminal point-like quantum moves to the right along an open helical trajectory of radius $R = \lambda / 2\pi$ with a speed of 1.414*c*.

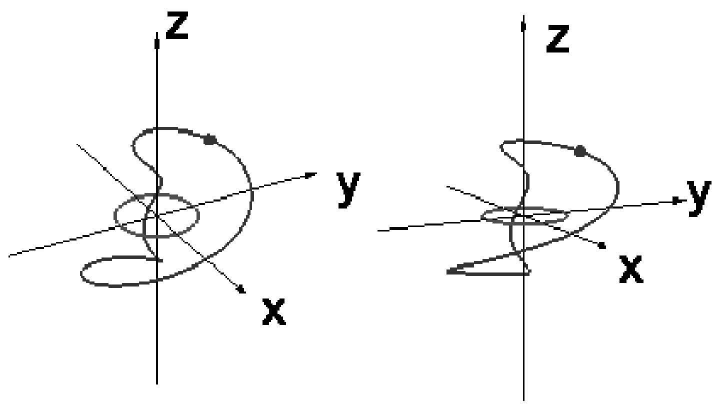
Some background facts are needed before describing the superluminal electron model. An electron has a rest mass *m* and a rest energy $E = mc^2$. A photon having this rest energy E will have a frequency given by $E = hf_0 = mc^2$ which gives $f_0 = mc^2 / h = 1.2 \times 10^{20}$ cycles per second. This frequency f_0 corresponds to the photon's angular frequency $\omega_0 = 2\pi f_0 = mc^2 / \hbar$ and the photon's wavelength given by $\lambda_C = h / mc$ which is called the Compton wavelength, which equals 2.4×10^{-12} m.

The superluminal electron model is composed of a charged point-like quantum moving in a closed double-looped helical trajectory. The circular axis of the closed double-looped helical trajectory has a double-looped length λ_C (each circumference of the double-looped circular axis is of length $\lambda_C / 2$) before the helical trajectory closes on itself. The circular axis therefore has a radius $R_0 = \dfrac{1}{2\pi} \lambda_C / 2 = \frac{1}{2} \hbar / mc$. The superluminal quantum rotates around its circular axis at angular frequency ω_0, with a constant helical radius $\sqrt{2} R_0$ while it moves forward with a speed *c* along its circular axis, just as the forward speed of the superluminal quantum in the photon model is *c*. Since the superluminal quantum follows a double-looped helical trajectory, its frequency for traveling forward around one circumference of its double-looped circular axis is $2 f_0$ and its corresponding angular frequency is $2\omega_0$. The helical trajectory of the electron's superluminal quantum can move forward in a clockwise or counterclockwise direction, corresponding to an electron or a positron (The turning direction for the electron or the positron is not specified in the electron model—this would require an experimental test of the superluminal electron model). The superluminal quantum moves along its helical trajectory at a variable speed since its trajectory has a circular axis. Its maximum speed is 2.515c.

If the circular axis of the double looped helix is in the x-y plane, the 3-D coordinates of the trajectory of the superluminal electron model are given (within a phase angle which would determine the position of the superluminal quantum along its trajectory at *t=0*) by

$$
\begin{aligned}
x(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\cos(2\omega_0 t)\\
y(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\sin(2\omega_0 t)\\
z(t) &= R_0\sqrt{2}\sin(\omega_0 t)
\end{aligned}
$$

where $R_0 = \frac{1}{2}\hbar / mc$ and $\omega_0 = mc^2/\hbar$. The 3-D trajectory of the superluminal quantum in the above electron model looks like this:



Two views of the superluminal model of the electron.
The charged superluminal point-like quantum moves in a closed double-looped helical trajectory whose circular axis is in the x-y plane. Its maximum speed is 2.515c.

Comparing the successes of the Dirac equation results with the superluminal model of the electron

The superluminal model of the electron has many of the physical characteristics of the Dirac electron, and has a 3-D physical structure as well. The Dirac electron properties and the corresponding superluminal electron model's properties are summarized below.

1) The Dirac electron was found to have a spin or angular momentum of $\frac{1}{2}\hbar$, the known spin of the electron.

   In the superluminal electron model, when the radius of the circular axis of the closed double-looped helix is set to equal $\frac{1}{2}\hbar / mc$, the calculated spin along the z-axis of the electron model, caused by the 3-D double-looped helical trajectory motion of the electric charge, is also $\frac{1}{2}\hbar$.

2) The Dirac equation's success in predicting the hydrogen frequency spectrum was partly due to its incorporating the effect on the spectrum of the electron's spin $\frac{1}{2}\hbar$ which came out of the Dirac equation solutions.

   The Dirac wave equation successfully incorporated the de Broglie wavelength/momentum relationship $\lambda = h / p$ of the electron. De Broglie derived this relationship assuming that the electron at rest had an internal frequency $\omega_0 = mc^2 / \hbar$ .This frequency is also contained in the superluminal electron model equations. Although de Broglie never mentioned that an electron could be composed of a photon-like object, the frequency ω_0 that de Broglie used to derive his wavelength/momentum relationship for moving electron is just the frequency of a photon having the same energy as the energy contained in the rest mass of an electron, according to Einstein's equation $E = mc^2$.

3) The Dirac electron is found to have a magnetic property or magnetic moment of $e\hbar / 2m$, which was the known value at that time of the electron's magnetic moment (it was later more precisely measured to be slightly higher than this by about 0.2%). In the superluminal electron model, when the radius of the double-looped helical trajectory is set to be $\sqrt{2}R_0 = \frac{\sqrt{2}}{2}\hbar / mc$, the calculated magnetic moment is also found to be exactly $e\hbar / 2m$, the same as for the Dirac electron. It is notable that only for this helical radius value $\sqrt{2}R_0$ that the root mean square (rms) values for the radius of the electron's double-looped helical trajectory in the x, y and z directions are all found to be exactly R_0.

4) The Dirac equation describes both the electron and the electron's antiparticle the positron, having equal mass and spin but opposite charge and opposite magnetic moment.

   In the superluminal electron model, the electron and the positron correspond to the two possible helicities or turning directions of the closed double-looped helical trajectory of an electron or a positron.

Problematical aspects of the Dirac electron compared with the superluminal electron model

1) The Dirac equation's negative energy electrons and his postulated electron sea prevented the acceptance of his equation at first. This interpretation was modified later in quantum electrodynamics by Hermann Weyl's group-theoretic approach to the symmetry structure of quantum mechanics, which put the electron and positron on an equal footing.

   The superluminal electron model with its electron and positron formed from the two possible helicities of the double-looped helical trajectory also put the electron and positron on an equal footing.

2) In the Dirac electron, the instantaneous speed of the electron is the speed of light *c*, while its average speed is less than *c*.

   In the superluminal electron model, although the speed of the electrically charged quantum is superluminal, the forward speed of this quantum along the circular axis of the double-looped helical trajectory is *c*. This corresponds to the forward speed *c* in the superluminal model of the photon, since the electron model is similar to a circulating photon model. Like the Dirac electron, the instantaneous position of the charged quantum in the superluminal electron model is different from the center of mass position of the electron (the average position of the quantum along its trajectory.) The average motion of the superluminal electron model would be the motion of the superluminal quantum's average position, which is $x=0$, $y=0$ and $z=0$ in the electron model's rest frame.

3) In the Dirac electron, the frequency of the oscillatory motion of the electric charge is the "jittery motion" frequency $2f_0 = 2mc^2/h$ or 2.5×10^{20} cycles per second.

   In the electron model, the circulatory frequency of the electron charge around the circular axis of the double-looped helical trajectory is also $2f_0 = 2mc^2/h$. The corresponding "jittery motion" angular frequency $2\omega_0 = 2\pi\times 2f_0 = 2mc^2/\hbar$ can be seen in the electron model's trajectory equations.

4) In the Dirac electron, the amplitude of the electric charge's oscillatory motion was found to be $\frac{1}{2}\hbar/mc$ or $1.9\times10^{-13}\,\mathrm{m}$.

   In the electron model the radius of the circular axis of the double-looped helical trajectory is also $\frac{1}{2}\hbar/mc$. Furthermore although the average position of the charge in the electron model's rest frame is zero in the x, y and z directions, the root mean square (rms) value of the x, y and z positions of the electric charge in its 3-D trajectory is also found to be $\frac{1}{2}\hbar/mc$. This is the value R_0 in the electron model's trajectory equations.

5) The Dirac electron returns to its original mathematical state after a 720 degree mathematical rotation.

   In the superluminal electron model, the superluminal quantum's trajectory closes after a 720 degree rotation about its *z*-axis while traveling along its double-looped circular axis. This 720 degree rotation is easily visualizable in the electron model. The superluminal quantum's trajectory doesn't close after a 360 degree rotation because it has completed only the first loop of its double-looped trajectory and at 360 degrees it is not in phase with its earlier trajectory.

6) In the Dirac electron at rest, instantaneous momentum is not conserved.

   In the superluminal electron model at rest, the superluminal quantum has a momentum which is constantly changing in both magnitude and direction at the very high frequency of $2f_0 = 2mc^2/h$ or 2.5×10^{20} cycles/sec and so, as in the Dirac electron, instantaneous momentum would not be conserved. It is only the electron model's average linear momentum which would be conserved.

7) The Dirac equation found an electric moment of magnitude $ie\hbar/2mc$. Because of the factor $i=\sqrt{-1}$, Dirac thought the electric moment was a physical effect, not a real result. As was quoted earlier from his book "Principles of Quantum Mechanics" (1983): "these extra terms involve some new physical effects, but since they are not real they do not lend themselves very directly to physical interpretation." But in science and engineering, a factor *i* in front of a physical quantity can also mean that the quantity is a physical vector at an angle of 90 degrees to another physical vector. In the Dirac electron the magnetic moment can have a component $e\hbar/2m$ in the *z* direction. The Dirac imaginary electric moment could correspond to an electric moment vector in the *x-y* plane, that is, perpendicular to the *z* direction. The value of an electric moment is calculated by multiplying the distance R of the charge from an axis by the magnitude of the electric charge: electric dipole moment $d = R\times e$.

   In the superluminal electron model, we can calculate the root mean square (rms) value of the electric dipole moment by multiplying the root mean square value of the electric charge's position by the charge *e*:

$$d_{rms} = R_{rms}\times e$$

   But we found previously that in the *x*, *y* and *z* directions, the root mean square (rms) value of the superluminal quantum trajectory's size was $R_{rms} = R_0 = \frac{1}{2}\hbar/mc$. Therefore the rms value of the electric dipole moment is given by:

$$\begin{aligned} d_{rms} &= R_{rms} \times e \\ &= (\tfrac{1}{2}\hbar / mc) \times e \\ &= e\hbar / 2mc \end{aligned}$$

which except for the imaginary factor *i* is just the magnitude of the electric moment $ie\hbar/2mc$ found by Dirac.

Martin Rivas (2005) comments on this imaginary electric moment that Dirac found: "But Dirac's equation also predicts another term $-\boldsymbol{d} \cdot \boldsymbol{E}$, of the coupling of an instantaneous electric dipole with the electric field. It is this oscillating electric dipole term that we believe is lacking in quantum mechanical wave equations. In general, the average value of this term in an electric field of smooth variation is zero. But in high intensity fields or in intergranular areas in which the effective potentials are low, but their gradients could be very high, this average value should not be negligible". Rivas shows that the electric moment of the Dirac electron could lead to interesting physical effects. For example, using a classical electron model ("...in the center of mass frame the particle describes a circle of radius $R_0 = \hbar/2mc$ ...") related to the Dirac electron, he shows that two electrons with parallel spins can form a bound pair if their centers of mass are separated by less than a Compton wavelength.

Conclusions

The Dirac equation of 1928 was very successful in accurately describing what was known about the fine structure of atomic spectra. It gave a theoretical basis for the electron's spin and magnetic moment. It led Dirac to predict the existence of previously undetected anti-matter. This prediction was soon confirmed for the anti-electron (the positron). But the Dirac electron could not be visualized and so remained a highly abstract entity, having a number of remarkable yet puzzling features.

The superluminal model proposed for the electron embodies a number of the Dirac electron's features in a visualizable quantitative model. The electron model's two possible internal helicities, corresponding to the electron and the positron, would seem to lend themselves to experimental testing of the electron model, for example by shining left and right circularly polarized gamma ray photons on electrons and looking for differences in the rates of scattering of these gamma ray photons from the electrons.

References

A.O. Barut (1991), "History of the Electron", in *The Electron: New Theory and Experiment, D. Hestenes* and A. Weingartshofer, ed., Kluwer Academic Publishers, Dordrecht, The Netherlands, p.109 .

B.L. Cline (1965), *The Questioners: Physicists and the Quantum Theory*, Thomas Y. Crowell Co., New York.

W.H. Cropper (2001) *Great Physicists*, Cambridge University Press, New York, p. 267.

P.A.M. Dirac (1928) "The quantum theory of the electron", *Proc. R. Soc.* **A 117,** 610-624, p. 619.

P.A.M. Dirac (1983) *The Principles of Quantum Mechanics*, 4th edition, Clarendon Press, Oxford, p. 265.

P.A.M. Dirac (1933) "Theory of electrons and positrons", http://nobelprize.org/physics/laureates/1933/dirac-lecture.pdf .

A. Eddington (1928) "Religion, Causation, Science and Mysticism", in *The Nature of the Physical World,* MacMillan Company.

R. Feynman (1965) Nobel lecture, at http://nobelprize.org/physics/laureates/1965/feynman-lecture.html .

R. Gauthier (2005) "Superluminal quantum models of the electron and the photon", at http://www.superluminalquantum.org/superluminalDec2005.pdf .

A. Pais (1994) "Playing with equations, the Dirac way", in *Reminiscences about a great physicist: Paul Adrien Maurice Dirac*, B.N. Kursunoglu and E.P. Wigner, eds., Cambridge University Press, 1987, p. 98.

M. Rivas (2005) "Kinematical formalism of elementary spinning particles", http://es.arxiv.org/PS_cache/physics/pdf/0509/0509131.pdf

J. Wheeler (1991), quoted in *Scientific American* 257, 6, 36

Richard Gauthier
richgauthier@gmail.com
http://www.superluminalquantum.org