Superluminal quantum models of the electron and the photon

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Abstract

A spatial model of a free electron (or a positron) is formed by a proposed superluminally circulating point-like charged superluminal quantum. The model includes the Dirac equation's electron spin $\frac{1}{2}\hbar$ and magnetic moment $e\hbar/2m$ as well as three Dirac equation measures of the electron's *Zitterbewegung* ("jittery motion"): a speed of light velocity *c*, a frequency of $2mc^2/h = 2.5\times10^{20}$ hz, and a size of $\frac{1}{2}\hbar/mc = 1.9\times10^{-13}$ m. The electron's superluminal quantum has a closed double-looped helical trajectory whose circular axis' double-looped length is one Compton wavelength h/mc. In the electron's rest frame, the equations for the superluminal quantum's position are:

$$
\begin{aligned}
x(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\cos(2\omega_0 t) \\
y(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\sin(2\omega_0 t) \\
z(t) &= R_0\sqrt{2}\sin(\omega_0 t)
\end{aligned}
$$

where $R_0 = \frac{1}{2}\hbar/mc$ and $\omega_0 = mc^2/\hbar$. The maximum speed of the superluminal quantum in the electron's rest frame is 2.516c. A photon is modeled by an uncharged superluminal quantum moving at $1.414c$ along an open 45-degree helical trajectory with radius $R = \lambda/2\pi$.

Introduction

Dirac's theory of the relativistic electron [1] did not include a model of the electron itself, and assumed the electron was a point-like particle. Schrödinger [2] analyzed the results of the Dirac equation for a free electron, and described a high-frequency *Zitterbewegung* which appeared to be due to the interference between positive and negative energy terms in the solution. Barut and Bracken [3] analyzed Schrödinger's *Zitterbewegung* results and proposed a spatial description of the electron where the *Zitterbewegung* would produce the electron's spin as the orbital angular momentum of the electron's internal system, while the electron's rest mass would be the electron's internal energy in its rest frame. Barut and Thacker [4] generalized Barut and Bracken's analysis of the internal geometry of the Dirac electron to a proper-time formalism. Hestenes [5-8] reformulated the Dirac equation through a mathematical approach (Clifford algebra) that brings out a geometric trajectory approach to understanding *Zitterbewegung* and to modeling the electron, such as identifying the phase of the Dirac spinor with the spatial angular momentum of the electron. A trajectory approach to the

Dirac theory has also been utilized by Bohm and Hiley [9], who describe the electron's spin angular momentum and its magnetic moment as due to the circulatory motion of a point-like electron. This dynamical approach to understanding *Zitterbewegung* has been elaborated by Holland [10].

The photon has been modeled geometrically, with several results quantitatively similar to those in the present superluminal quantum model of the photon, though obtained through different approaches. Ashworth [11] obtained a superluminal helical photon model whose radius is $\lambda / 2\pi$, the same as in the proposed superluminal quantum model of the photon, from classical mechanics considerations. Kobe [12], in a *Zitterbewegung* approach to the photon, also obtained the same quantitative result for the radius of a classical helical model of the photon based on quantum mechanical considerations, as independently did Sivasubramanian *et al*.[13], whose helical photon model, also based on a *Zitterbewegung* approach to the photon, is explicitly internally superluminal.

A unified superluminal quantum approach to modeling the electron and the photon

The present approach is a unified approach to modeling both the electron and the photon with superluminal helical trajectories. The electron model has several features of the Dirac electron's *Zitterbewegung*. Point-like entities are postulated called superluminal quanta (to distinguish them from electrons and photons themselves.) They have an energy *E*, with its associated frequency *f* and angular frequency $\omega = 2\pi f$, an instantaneous momentum $\vec{P}$ with its associated wavelength λ and wave number $k = 2\pi / \lambda$, and an electric charge (in the case of the electron). One superluminal quantum forms a photon or an electron. Superluminal quanta move in helical trajectories, which may be open (for a photon) or closed (for an electron). Movement of the superluminal quantum along its trajectory produces an electron or a photon. The type of helical trajectory determines which particle is produced.

The energy of a superluminal quantum composing either a photon or electron is $E = \hbar\omega$ where ω is the angular frequency of rotation of the superluminal quantum along its helical trajectory, whether the trajectory is open or closed. The momentum vector $\vec{P}$ of a superluminal quantum is directed tangentially along its helical trajectory. The total momentum $\vec{P}$ has a component $p = \hbar k$ that is projected parallel to the helical axis around which the superluminal quantum is moving. $\vec{P}$ changes in direction as the superluminal quantum moves along its helical trajectory. In the photon model, $\vec{P}$ has constant magnitude but changes in direction as the superluminal quantum travels along an open helix having a constant wavelength and curvature. The projected momentum *p* along the helical axis will be constant in magnitude. But in the electron model, $\vec{P}$ changes in magnitude as well as direction with the changing curvature of the closed helical trajectory along which the electron's superluminal quantum travels. Here $\vec{P}$'s projected magnitude *p* along the helical axis, as well as its corresponding wavelength, will also vary in magnitude.

The longitudinal component of the velocity of a superluminal quantum along its helical axis, whether the helix is open in the photon model or closed in the electron model, is postulated to always be exactly the speed of light *c*. In the photon model, the superluminal quantum moves at a constant superluminal speed along an open helical trajectory with a straight axis. In the electron model, the speed of the superluminal quantum varies along the closed helical trajectory and the helical trajectory's axis is circular.

The following superluminal quantum models of a photon and an electron will illustrate the superluminal quantum's properties more concretely.

The superluminal quantum model of the photon

A photon is modeled as a superluminal quantum traveling along an open helical trajectory of radius R and pitch (wavelength) λ. The trajectory makes an angle θ with the forward direction. In this helical trajectory, these three quantities are related geometrically by $\tan\theta = 2\pi R/\lambda$. The superluminal quantum model for a photon of any wavelength is found to have the following properties:

1) The forward angle θ of the helical trajectory is 45°.
2) The radius of the superluminal quantum's helical trajectory is $R = \lambda / 2\pi$
3) The speed of the superluminal quantum is $\sqrt{2}c = 1.414..c$ along its helical trajectory.

These three results are derived below. An image of the superluminal quantum model of a photon is shown in Figure 1.

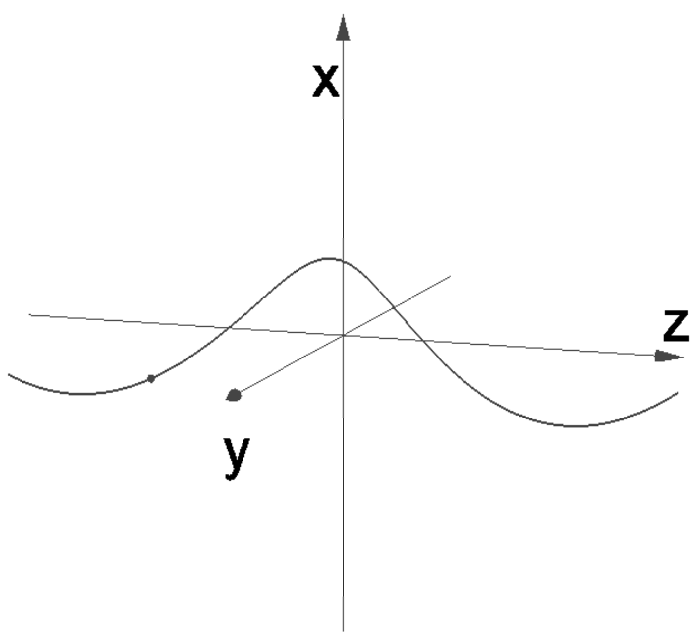

Figure 1. Superluminal model of a photon. A superluminal quantum moves along its 45-degree open helical trajectory. The radius of the helix is $R = \lambda / 2\pi$. The speed of the superluminal quantum along its trajectory is 1.414c.

The superluminal quantum, with total momentum $\vec{P}$ directed along its helical trajectory, has a longitudinal component of momentum $P\cos(\theta)$ determined by the wavelength λ of the helix, and a transverse component of momentum $P\sin(\theta)$ that is used to calculate the angular momentum or spin of the photon. The superluminal quantum's longitudinal component of momentum is

$$P\cos(\theta) = h / \lambda \tag{1}$$

the experimental linear momentum of a photon. $\vec{P}$'s transverse component of momentum $P\sin\theta$, acting at the helical radius R from the helical axis, produces an angular momentum or spin S whose longitudinal magnitude in the direction the photon is moving (or in the opposite direction depending on the helicity of the trajectory) is

$$S = RP\sin(\theta) = h / 2\pi \tag{2}$$

which is the experimental spin or angular momentum of the photon. Combining equations (1) and (2) gives

$$\sin(\theta) / \cos(\theta) = \tan(\theta) = \lambda / 2\pi R \tag{3}$$

Now consider the helical geometry. As the superluminal quantum advances along the helix a distance λ (one wavelength) in the longitudinal direction, the superluminal quantum travels a transverse distance $2\pi R$, i.e. once around the circle of radius R of the helix. From the way the helical trajectory's forward angle θ is defined, we have

$$\tan(\theta) = 2\pi R / \lambda \tag{4}$$

We now have two equations (3) and (4) for $\tan(\theta)$. Setting them equal gives

$$\tan(\theta) = 2\pi R / \lambda = \lambda / 2\pi R \tag{5}$$

This will only be true when

$$\lambda = 2\pi R \tag{6}$$

that is, when

$$R = \lambda / 2\pi \tag{7}$$

This result implies that $\tan\theta = 1$ and therefore

$$\theta = 45^{\circ} \tag{8}$$

These results for the superluminal quantum model of the photon are true for any wavelength. Since the longitudinal velocity component of the photon's superluminal quantum along its helical axis is postulated to be *c*, the velocity of the photon's superluminal quantum along its helical trajectory is

$$v = c/\cos(45^{\circ}) = c/(\sqrt{2}/2) = 1.414..c \tag{9}$$

Using these results, for a right-handed photon traveling in the +*z* direction, the equations for the trajectory of the superluminal quantum (neglecting a possible phase factor) are:

$$\begin{aligned} x(t) &= \frac{\lambda}{2\pi}\cos(\omega t) \\ y(t) &= \frac{\lambda}{2\pi}\sin(\omega t) \\ z(t) &= ct \end{aligned} \tag{10}$$

where $\omega = 2\pi f = 2\pi c/\lambda$ is the angular frequency of the photon, f is the photon's frequency in cycles per second and λ is the photon's wavelength. In the superluminal photon model, λ is the distance along the helical axis corresponding to one rotation of the superluminal quantum along its helical trajectory.

Similarly, for this right-handed photon, the equations for the components of the momentum of the superluminal quantum along its trajectory are

$$\begin{aligned} p_x(t) &= -\frac{h}{\lambda}\sin(\omega t) \\ p_y(t) &= \frac{h}{\lambda}\cos(\omega t) \\ p_z(t) &= \frac{h}{\lambda} \end{aligned} \tag{11}$$

The *x* and *y* components of momentum are 90 degrees out of phase with the *x* and *y* position values.

The Heisenberg uncertainty principle and the superluminal quantum model of the photon

With the superluminal quantum model of the photon, the superluminal quantum would be what is actually detected when a single photon is detected in an experiment. Suppose a photon is traveling in the +z direction. Because of its varying position and momentum components as it moves along its trajectory, a range of values of its *x* and *y* components of position and momentum would be detected when various photons traveling in the +*z* direction are measured successively.

A remarkable aspect of the superluminal model of the photon is that the superluminal quantum's position and momentum components are found to be quantitatively closely related to the Heisenberg uncertainty principle. This principle says that there is a fundamental limitation on the accuracy of simultaneously measuring two related physical properties, such as the position and momentum, of an elementary particle or other physical object. Greater accuracy in measuring one of the two properties entails a corresponding lesser accuracy in measuring the other related property. The Heisenberg uncertainty relationship for the *x* coordinate of a particle is stated precisely as $\Delta x\ \Delta p_x\ \geq h/4\pi$, where Δx is the standard error (the square root of the statistical variance) in measuring the position of the particle along the *x* direction , Δp_x is the standard error in measuring the particle's momentum along the same *x* dimension, "$\geq$" means "is greater than or equal to", and *h* is Planck's constant, an extremely small quantity. How does the Heisenberg uncertainty relation apply to detecting a photon in the superluminal photon model?

Consider detecting the *x* position and the p_x component of the momentum of the superluminal quantum in a photon that is moving horizontally in the +*z* direction. From the results for the superluminal quantum model of the photon obtained earlier, the radius of the superluminal quantum's helical trajectory is $R = \dfrac{\lambda}{2\pi}$. In this example, where the average value of $x(t)$ over a cycle is zero, the standard error Δx is the root mean square (rms) of *x*, that is the square root of the average value of $\left[x(t)\right]^2$ over one cycle. Similarly Δp_x, the standard error or rms of $p_x(t)$, is the square root of the average value of $\left[p_x(t)\right]^2$ over one cycle. So $\left[x(t)\right]^2 = (\dfrac{\lambda}{2\pi})^2\cos^2(\omega t)$ while $\left[p_x(t)\right]^2 = (\dfrac{h}{\lambda})^2\sin^2(\omega t)$. The average value of $\cos^2(\omega t)$ and of $\sin^2(\omega t)$ over a cycle is $\dfrac{1}{2}$ for each. So the average value of $\left[x(t)\right]^2$ is $\dfrac{1}{2}(\dfrac{\lambda}{2\pi})^2$ and the average value of $\left[p_x(t)\right]^2$ is $\dfrac{1}{2}(\dfrac{h}{\lambda})^2$. Therefore $\Delta x = \sqrt{\dfrac{1}{2}(\dfrac{\lambda}{2\pi})^2} = \dfrac{1}{\sqrt{2}}\dfrac{\lambda}{2\pi}$ while $\Delta p_x = \sqrt{\dfrac{1}{2}(\dfrac{h}{\lambda})^2} = \dfrac{1}{\sqrt{2}}\dfrac{h}{\lambda}$. Multiplying Δx by Δp_x we get $\Delta x \Delta p_x = (\dfrac{1}{\sqrt{2}}\dfrac{\lambda}{2\pi})(\dfrac{1}{\sqrt{2}}\dfrac{h}{\lambda}) = \dfrac{h}{4\pi}$ for the superluminal quantum model of the electron.

Comparing this result with the Heisenberg uncertainty relation $\Delta x \Delta p_x \geq \dfrac{h}{4\pi}$ we see that the uncertainty product of the transverse or *x* components of position and momentum for the superluminal quantum in the photon model is exactly the minimum value allowed by the Heisenberg uncertainly relation.

The same quantitative results are found for Δy and Δp_y, the rms values for other transverse components of position and momentum of the superluminal quantum. So

$\Delta y \Delta p_y \geq \frac{h}{4\pi}$. In the case of the photon model's longitudinal components *z* and p_z, the value of p_z is h/λ, which is a constant for this photon, so $\Delta p_z = 0$. An open helix in the *z* direction corresponds to an infinitely long trajectory in the *z* direction, so the position of the superluminal quantum along its trajectory in the *z* direction would be completely uncertain, i.e. $\Delta z = \infty$. In this case $\Delta z \Delta p_z = 0 \times \infty$, which is undefined. This also corresponds to the Heisenberg uncertainty relation: when the momentum component of an object is completely certain, its corresponding position component is completely uncertain.

The above results for the superluminal quantum model of the electron as compared with the Heisenberg uncertainty relation could merely be a coincidence. Or there could be a deeper reason, which is not yet apparent, for these identical results.

Any real photon will have a finite value of uncertainty in the coordinates of both its position and its momentum. A photon, until it is detected, is described quantum mechanically by a mathematical superposition of position states or their corresponding momentum states, each corresponding to a particular wave function with a particular amplitude and phase. This total quantum wave function describing the photon is then related to the probability of detecting the photon in the regions where the total wave function is non-zero. The superluminal photon model seems to be consistent with the quantum mechanical interpretation of the photon and the Heisenberg uncertainty principle.

The superluminal quantum model of the electron

Besides having the electron's experimental spin value and the magnetic moment of the Dirac electron, the superluminal quantum model of the electron, described below, quantitatively embodies the "*Zitterbewegung*", the small and rapid oscillatory motion of the electron that is predicted by the Dirac equation but which has not been experimentally observed.

Zitterbewegung refers to the Dirac equation's predicted rapid oscillatory motion of an electron than adds to its center-of-mass motion. No size or spatial structure of the electron has so far been observed experimentally. High energy electron scattering experiments by Bender *et al*. [14] have put an upper value on the electron's size at about 10^{-18} m. Yet Schrödinger's *Zitterbewegung* results suggest that the electron's rapid oscillatory motion has a magnitude of $R_{zitt} = ½\hbar / mc$ or 1.9×10^{-13} m and an angular frequency of $\omega_{zitt} = 2mc^2/\hbar = 1.6 \times 10^{21}/\mathrm{sec}$, twice the angular frequency $\omega_0 = mc^2/\hbar$ of a photon whose energy is that contained within the rest mass of an electron. Furthermore, in the Dirac solution the electron's instantaneous speed is *c*, although experimentally observed electron speeds are always less than *c*. An acceptable model of the electron would presumably contain these *Zitterbewegung* properties of the Dirac electron.

In the present superluminal quantum model of the electron, the electron is composed of a charged superluminal point-like quantum moving along a closed, double-looped helical trajectory in the electron model's rest frame, that is, the frame where the superluminal quantum's trajectory closes on itself. (In an moving inertial reference frame, the superluminal quantum's double-looped helical trajectory will not exactly close on itself.) The superluminal quantum's trajectory's closed helical axis' radius is set to be $R_0 = \frac{1}{2}\hbar / mc = 1.9 \times 10^{-13}\,\mathrm{m}$ and the helical radius is set to be $R_{helix} = \sqrt{2} R_0$. The superluminal quantum electron model structurally resembles a superluminal quantum photon model of angular frequency $\omega_0 = mc^2 / \hbar$, wavelength $\lambda_C = h / mc$ (the Compton wavelength) and wave number $k = 2\pi / \lambda_C$ that, instead of having a straight axis, moves in a circular pattern to form a double-looped helical trajectory having a circular axis of circumference $\lambda_C / 2$. After following its helical trajectory around this circular axis once, the electron's superluminal quantum's trajectory is 180^{o} out of phase with itself and doesn't close on itself. But after the superluminal quantum traverses its helical trajectory around the circular axis a second time, the superluminal quantum's trajectory is back in phase with itself and closes upon itself. The total longitudinal distance along its circular axis that the circulating superluminal quantum has traveled before its trajectory closes is λ_C.

In its rest frame, the electron's superluminal quantum carries energy $E = \hbar\omega_0 = mc^2$. Unlike the photon's superluminal quantum which is uncharged, the electron's superluminal quantum carries the electron's negative charge –*e*.

The above closed, double-looping helical spatial trajectory for the superluminal quantum in the electron model is given in rectangular coordinates by

$$
\begin{aligned}
x(d) &= R_0(1+\sqrt{2}\cos(2\pi d/\lambda_C))\cos(4\pi d/\lambda_C) \\
y(d) &= R_0(1+\sqrt{2}\cos(2\pi d/\lambda_C))\sin(4\pi d/\lambda_C) \\
z(d) &= R_0\sqrt{2}\sin(2\pi d/\lambda_C)
\end{aligned}
\tag{12}
$$

where $R_0 = \frac{1}{2}\hbar / mc$ is the radius of the circle which is the axis of the double-looped helical trajectory, $\lambda_C = h / mc$, and *d* is the distance forward along the circular axis that the superluminal quantum has moved while following its helical trajectory. This forward speed is postulated to be *c*, consistent with the Dirac electron's *Zitterbewegung* results, so in the superluminal quantum's trajectory above, $d = ct$.

Note that when $d = \frac{1}{2}\lambda_C$ (at one traversal of the circular axis of the closed helical trajectory), the term $2\pi d / \lambda_C$ in (12) above has value π. So $\cos(2\pi d / \lambda_C)$ and $\sin(2\pi d / \lambda_C)$ are only 180^{o} into their cycles, which only reaches 2π or 360^{o} when $d = \lambda_C$ (at two traversals of the circular axis of the closed helical trajectory). The second term $4\pi d / \lambda_C$ in (12) above is in phase at both $d = \frac{1}{2}\lambda_C$, which gives the phase 2π (in phase), and $d = \lambda_C$, which gives the phase 4π (in phase), but all of the sine and cosine terms in the equations in (12) have to be in phase for the helical trajectory to close on itself.

Since $d = ct$, the term $\cos(2\pi d / \lambda_C)$ in (12) becomes

$$\cos(2\pi d / \lambda_C) = \cos(2\pi ct / \lambda_C) = \cos(2\pi f_0 t) = \cos(\omega_0 t) \tag{13}$$

and similarly for the other terms in (12). So the position with time of the superluminal quantum in the electron model becomes

$$\begin{aligned} x(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\cos(2\omega_0 t) \\ y(t) &= R_0(1+\sqrt{2}\cos(\omega_0 t))\sin(2\omega_0 t) \\ z(t) &= R_0\sqrt{2}\sin(\omega_0 t) \end{aligned} \tag{14}$$

where $R_0 = \frac{1}{2}\hbar / mc$ and $\omega_0 = mc^2 / \hbar$. These equations correspond to a left-handed photon-like object of wavelength λ_C , circulating counterclockwise (as seen above from the +z axis) in a closed double loop. Two images from different perspectives of the superluminal quantum model of an electron are shown in Figure 2.

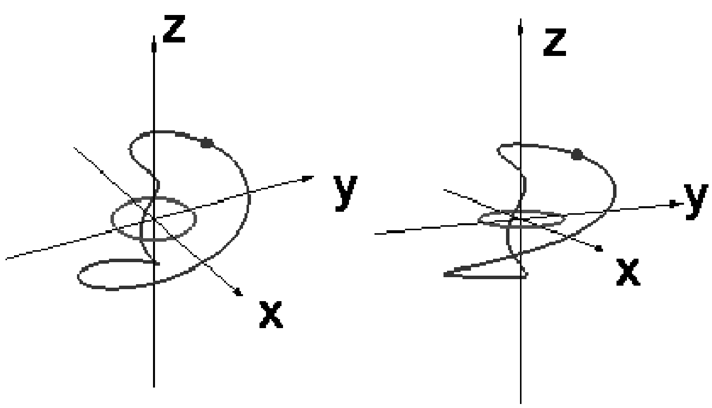

Figure 2. Superluminal model of an electron. Two views of a superluminal quantum moving along its closed double-looped helical trajectory. The circle in the x-y plane of radius $R_0 = \frac{1}{2}\hbar / mc$ =1.9 x 10^{-13} *m is the axis of the closed helix. The maximum speed of the superluminal quantum in the electron's rest frame is 2.516c .*

Differentiating the position coordinates of the superluminal quantum in equation (14) with respect to time, we get its velocity components to be

$$
\begin{aligned}
v_x(t) &= -c[(1+\sqrt{2}\cos\omega_0 t)\sin 2\omega_0 t + \frac{\sqrt{2}}{2}\cos 2\omega_0 t \sin\omega_0 t] \\
v_y(t) &= c[(1+\sqrt{2}\cos\omega_0 t)\cos 2\omega_0 t - \frac{\sqrt{2}}{2}\sin 2\omega_0 t \sin\omega_0 t] \\
v_z(t) &= c\frac{\sqrt{2}}{2}\cos\omega_0 t
\end{aligned} \tag{15}
$$

From equation (14), at $t = 0$, the superluminal quantum's position coordinates are $x(0) = R_0(1+\sqrt{2})$, $y(0) = 0$, and $z(0) = 0$. At this time $t = 0$ the quantum is at its maximum speed of 2.516*c*. This is found by getting the velocity components at $t = 0$ from equation (15) : $v_x(0) = 0$, $v_y(0) = c(1+\sqrt{2}) = 2.414c$, and $v_z(0) = c\frac{\sqrt{2}}{2} = .707c$ and using $v = \sqrt{v_x(t)^2 + v_y(t)^2 + v_z(t)^2}$. At $t = T_0 = 2\pi/\omega_0 = h/mc^2$ the quantum completes a full cycle around its closed trajectory and its speed *v* again reaches 2.516*c*. But the quantum's speed is variable along its closed double-looped helical trajectory, even passing below the speed *c* during part of its trajectory. For example, at the halfway point in its trajectory when the circulating quantum is nearer the *z*-axis, (at $t = T_0/2 = \pi/\omega_0 = h/2mc^2$) its speed as found from $v_x(t)$, $v_y(t)$ and $v_z(t)$ is $v = .819c$. And twice when quantum passes through the z-axis while traveling along its closed helical trajectory, it reaches its minimum speed along its helical trajectory of $c/\sqrt{2}$ or .707*c*.

The circulating quantum spends approximately 56% of its time (measured in the electron model's rest frame) traveling superluminally along its trajectory and 44% of its time traveling subluminally. The quantum twice passes through the speed value *c* while completing one closed helical trajectory. This passage of the quantum from superluminal speeds through *c* to subluminal speeds and back again to superluminal speeds is not a problem from a relativistic perspective. This is because it is the point-like electric charge -*e* that is moving at these speeds and not the average center of mass/energy of the electron model, which remains at rest in the electron model's rest frame.

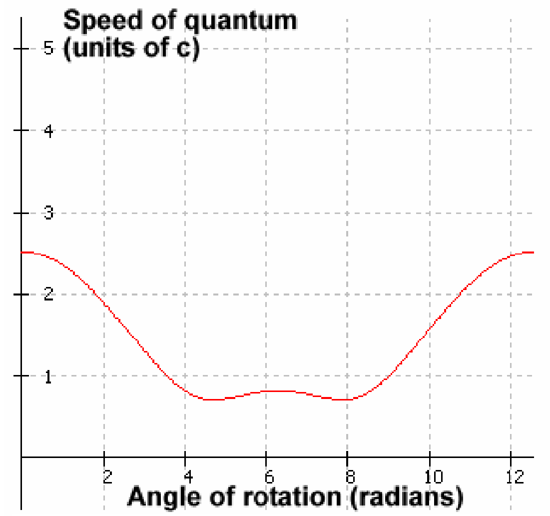

Figure 3. Speed of the quantum along its double looped helical trajectory. One complete trajectory requires a rotation about the z-axis of 4π *or 12.57 radians. The maximum speed of the quantum is 2.516c. Its minimum speed is .707c . The quantum is superluminal 56% of the time.*

Similarities between the Dirac equation's free electron solution and the superluminal quantum electron model

The superluminal quantum model of the electron share a number of quantitative and qualitative properties with the Dirac equation's electron with *Zitterbewegung*:

1) **The *Zitterbewegung* internal frequency of** $\omega_{zitt} = 2mc^2/\hbar = 2\omega_0$ **.**

The superluminal quantum's trajectory in equation (14) is defined by both the frequencies ω_0 and $2\omega_0$. ω_0 is the angular frequency of a photon whose energy is mc^2 .

2) **The *Zitterbewegung* radius** $R_0 = \frac{1}{2}\hbar/mc = R_{zitt}$ **.**

R_0 in equation (14) is the radius of the circular axis of the closed double-looped helical trajectory of the superluminal quantum model. Furthermore R_0 was found to be the root mean square (rms) value for the *x*, *y* and *z* coordinates of the electron's superluminal quantum's trajectory described by equation (12). These three rms values of R_0 are only found if the radius of the closed helical trajectory described in equation (14) is $\sqrt{2}R_0$, which was the value required to give the superluminal electron model the required magnetic moment value $M_z = -e\hbar/2m$.

3) **The *Zitterbewegung* speed-of-light result for the electron.**

The Dirac equation has eigenvalue solutions of $\pm c$ for the velocity of the electron (and the positron). The longitudinal component along the circular axis of the superluminal quantum's closed helical trajectory was postulated to be *c*, just as in the superluminal quantum model of the photon. This internal speed *c* is incorporated into the superluminal quantum trajectory equation (12) for the electron model as $d = ct$, where *d* is the forward distance along the helical axis traveled by the superluminal quantum as it follows its helical trajectory. The maximum speed of the electron's superluminal quantum itself is found from the trajectory equations (14) to be 2.516*c*.

4) **The prediction of the electron's antiparticle.**

The two possible helicities of the superluminal quantum's closed helical path correspond to an electron and a positron. (The existence of the positron is implied by the results of Dirac's equation.). By reversing the helicity for the superluminal quantum described by the above closed double-looped helix in equation (14), whose left-turning superluminal quantum corresponds to a left-handed photon, the corresponding anti-particle's superluminal quantum would be formed, which would correspond to a right-handed photon. A charge of +e would have to be supplied to the positron's superluminal quantum for symmetry. The superluminal quantum electron model does not however predict whether the circulating left-handed superluminal quantum trajectory described by equation (14) is actually associated with an electron or a positron. This could be tested by an experiment.

The relationship between charge and helicity in the superluminal quantum electron and photon models is suggestive of a deeper relationship between helicity, spin and charge. The superluminal quantum photon model has an open helical trajectory, spin $\hbar$ and no charge. The superluminal quantum electron model has a closed double-looped helical trajectory, spin $\frac{1}{2}\hbar$ and a negative charge.

5) **The calculated spin of the electron.**

The value of R_0 in the electron model's superluminal quantum trajectory in equation (12) is chosen to give the electron's experimental value of spin $\hbar/2$, the spin value also found from the Dirac equation.

The calculation of the electron's angular momentum or spin in the superluminal quantum model of the electron is complicated by the varying radial distance and the correspondingly varying wavelength and therefore momentum of the circulating superluminal quantum along its closed helical trajectory. The instantaneous angular momentum of a circulating object with momentum $\vec{P}$ at a distance $\vec{R}$ from a rotational axis is

$$\vec{S} = \vec{R} \times \vec{P} \tag{16}$$

In the superluminal quantum model for the electron, the superluminal quantum's instantaneous position and momentum can be described by a radial vector $\vec{R}$ and a momentum vector $\vec{\mathrm{P}}(\vec{\mathrm{R}})$. The magnitude of $\vec{R}$, which can be obtained from the electron's superluminal quantum trajectory equation (14), varies with the superluminal quantum's position along its trajectory. The magnitude and direction of $\vec{\mathrm{P}}(\vec{\mathrm{R}})$, the superluminal quantum's instantaneous momentum along its trajectory, are related to the instantaneous wavelength λ of its trajectory.

The instantaneous wavelength λ at a point on the double-looped helical trajectory in equation (14) can be defined consistently as twice the circumference of the circle that is parallel to the *x*-*y* plane and centered on the *z*-axis, having radius *R* and passing through that point on the trajectory. The radial distance *R* from the *z*-axis to that point is then directly proportional to the instantaneous wavelength λ at that point:

$$R = K\lambda \tag{17}$$

where *K* is the proportionality constant. From equation (12) and the above definition of the instantaneous value of λ ,

$$K = R_0 / \lambda_C = 1/4\pi \tag{18}$$

Substituting equation (18) into equation (17), the relation of *R* to the instantaneous wavelength λ at a point on the trajectory is

$$R = \lambda / 4\pi \tag{19}$$

Now *p*, the magnitude of the longitudinal component of $\vec{\mathrm{P}}(\vec{\mathrm{R}})$ along the above-defined circle of radius *R*, is postulated to be inversely proportional to the instantaneous wavelength λ at that point, that is,

$$p = h / \lambda \tag{20}$$

the same as a photon's momentum relationship with its wavelength. This is consistent with the photon model because the superluminal quantum model of the electron is basically a charged circulating superluminal quantum model of a photon.

It is this component *p* of the superluminal quantum's total momentum $\vec{\mathrm{P}}(\vec{\mathrm{R}})$ that contributes to S_z , the free electron's spin or angular momentum. By combining equations (16), (19) and (20), the instantaneous value of the angular momentum S_z at any point along the closed helical trajectory is given by

$$\begin{aligned} S_z &= Rp = (\lambda / 4\pi)(h / \lambda) \\ &= h / 4\pi \\ &= \tfrac{1}{2}\hbar \end{aligned} \tag{21}$$

which is the value of spin of the electron found from the Dirac equation, and which is also experimentally correct. Despite the variation in $\vec{R}$ and $\vec{\mathrm{P}}(\vec{\mathrm{R}})$ of the superluminal quantum along its closed helical trajectory, the instantaneous spin S_z of the electron remains constant.

6) **The calculated magnetic moment of the electron.**

For the closed, double-looped helical trajectory of the charged superluminal quantum given in equation (14) the electron's magnetic moment M_z is found from the total magnetic moment $\vec{M}$ of a 3-dimensional loop carrying a current I caused by the motion of the point charge –e around its closed trajectory given in equation (14) in time period $T = h/mc^2 = 2\pi/\omega_o$:

$$\vec{M} = \frac{I}{2}\int_{t=0}^{T} \vec{r}(t) \times d\vec{r}(t) \quad \text{where } T = h/mc^2$$

$$\vec{M} = \frac{I}{2}\int_{\theta=0}^{2\pi} \vec{r}(\theta) \times \frac{d\vec{r}(t)}{dt}\frac{dt}{d\theta}d\theta$$

$$\vec{M} = \frac{I}{2}\int_{\theta=0}^{2\pi} \vec{r}(\theta) \times \vec{v}(\theta)\frac{1}{\omega_0}d\theta \quad \text{since } \theta = \omega_0 t$$

$$\vec{M} = \frac{1}{2}\times\frac{-e\omega_0}{2\pi}\int_{\theta=0}^{2\pi} \vec{r}(\theta) \times \vec{v}(\theta)\frac{1}{\omega_0}d\theta \quad \text{since } I = \frac{-e\omega_0}{2\pi}$$

$$\vec{M} = \frac{1}{2}\times\frac{-e\omega_0}{2\pi}\int_{\theta=0}^{2\pi} \vec{r}(\theta) \times \vec{v}\,\frac{1}{\omega_0}d\theta$$

$$\vec{M} = \frac{-e}{4\pi}\int_{\theta=0}^{2\pi} \vec{r}(\theta) \times \vec{v}(\theta)d\theta$$

$$M_z = \frac{-e}{4\pi}\int_{\theta=0}^{2\pi} (R_x(\theta)V_y(\theta) - R_y(\theta)V_x(\theta))d\theta \tag{22}$$

The superluminal quantum electron's magnetic moment's value in equation (22) was set to be $M_z = -e\hbar/2m$, the value of the Bohr magneton M_B (corresponding to the magnetic moment of the Dirac electron). Using the closed double-looped helical trajectory structure of equation (12) and solving equation (22) for the unknown value R for the radius of the superluminal quantum's closed helical trajectory that would yield $M_z = -e\hbar/2m$, gave $R = \sqrt{2}R_0$ where $R_0 = \frac{1}{2}\hbar/mc$. It is this radius $R = \sqrt{2}R_0$ that was then included in equations (12) and (14) to obtain the trajectory of the superluminal quantum model of the electron.

Using the same value helical radius value $R = \sqrt{2}R_0$ that gives $M_z = -e\hbar/2m = -M_B$, the values for M_x and M_y components of M for the superluminal electron model can be found from

$$M_x = \frac{-e}{4\pi}\int_{\theta=0}^{2\pi} (R_y(\theta)V_z(\theta) - R_z(\theta)V_y(\theta))d\theta = 0$$
$$M_y = \frac{-e}{4\pi}\int_{\theta=0}^{2\pi} (R_z(\theta)V_x(\theta) - R_x(\theta)V_z(\theta))d\theta = .25M_B \tag{23}$$

The values of M_x and M_y depend on the way the *x* and *y* components of the electron model's helical trajectory are defined in equation (14). A rotation of the *x* and *y* components of these equations by +90 degrees around the *z-axis* would give $M_x = -.25M_B$ and $M_y = 0$, while leaving unchanged $M_z = -M_B$.

7) **The electron's motion is the sum of its center-of-mass motion and its *Zitterbewegung*, with the motion of the electron's charge distinct from the motion of its center of mass.**

For the Dirac electron this is described in [3] where the *Zitterbewegung* is obtained by solving the Heisenberg equations of motion for the position operator $\vec{\mathrm{x}}(t)$ using the Dirac Hamiltonian $H = m\gamma^0 + \vec{\mathrm{p}}\cdot\vec{\alpha}$ $(\hbar = c = 1)$. The coordinate operator $\vec{\mathrm{x}}(t)$ contains a "center-of-mass" part $\vec{\mathrm{X}}(t) = H^{-1}\vec{\mathrm{p}}t + \vec{a}$ $(\vec{a} = \text{constant vector})$ that moves with a uniform velocity, and an oscillatory part $\vec{\xi}(t) = \frac{i}{2}[\vec{\alpha}(0) - H^{-1}\vec{p}]H^{-1}e^{-2iHt}$ which is the *Zitterbewegung*. Therefore in the Dirac free electron solution, $\vec{\mathrm{x}}(t) = \vec{\mathrm{X}}(t) + \vec{\xi}(t)$, where $\vec{\mathrm{x}}(t)$ is interpreted as the instantaneous position of the electron's point-like charge. This point-like charge oscillates rapidly according to $\vec{\xi}(t)$ about the center of mass $\vec{\mathrm{X}}(t)$.

The center-of-mass motion $\vec{\mathrm{X}}(t)$ is the subluminal linear motion of the Dirac free electron, while the *Zitterbewegung* $\vec{\xi}(t)$ is the electron's speed-of-light oscillatory motion. In the superluminal quantum model of the electron, equation (14) , corresponding to $\vec{\xi}(t)$, describes the trajectory of the electron's superluminal charged quantum, whose longitudinal component of velocity around its closed helical trajectory's circular axis is the speed of light *c*. Equation (14), representing the motion of the superluminal charge in the electron's rest frame, can have an added linear position component *vt* (corresponding to the electron's center-of-mass velocity *v*) in the *z* direction which would correspond to $\vec{\mathrm{X}}(t)$ above.

The double-looping helical trajectory, which closes on itself exactly in the rest frame, would therefore not close exactly in a moving frame due to the small average displacement of the charged quantum in the z direction during each cycle of the superluminal quantum along its helical trajectory. The position of the electric charge in the Dirac electron is not the same as the position of the electron's center of mass. This is also the case in the superluminal quantum model, where the electron's charge is moving with the superluminal quantum, but the electron as a whole, with its total energy content and average center-of-mass position, can only move subluminally.

8) **The non-conservation of linear momentum in the *Zitterbewegung* of a free electron.**

In the oscillatory *Zitterbewegung* of a free electron, linear momentum is not conserved, a result first pointed out by de Broglie [15]. In the *Zitterbewegung*'s quantum dynamics the expectation value $< dp_k / dt >$ does not equal zero, even in the absence of an applied force F_k. This lack of conservation of linear momentum in the *Zitterbewegung* is also the case in the rest frame of the superluminal quantum model of the electron, due to the rapid changing in magnitude and direction of the superluminal quantum's linear momentum vector $\vec{P}$ as the superluminal quantum moves along its closed helical trajectory. But violations of the conservation of energy can occur in quantum electrodynamics if the time interval in which the violation occurs is shorter than the minimum time permitted for experimental observations by the Heisenberg uncertainty relations. In the same way, violations of conservation of linear momentum in the Dirac electron's *Zitterbewegung* , as well as in the present superluminal model of the electron, may be similarly permitted within the range of the size of the electron's *Zitterbewegung* amplitude $R_0 = \frac{1}{2}\hbar / mc = 1.9\times 10^{-13}\,\mathrm{m}$.

The Heisenberg uncertainty principle and the superluminal quantum model of the electron

The Heisenberg uncertainty relationship for the *x* coordinate of a particle is stated precisely as $\Delta x\ \Delta p_x\ \geq h / 4\pi$, where Δx is the standard error (the square root of the statistical variance) in measuring the position of the particle along the *x* direction , Δp_x is the standard error in measuring the particle's momentum along the same *x* dimension, How does the Heisenberg uncertainty relation apply to detecting an electron in the superluminal quantum model of the electron?

Using the equations (14) for the *x*, *y* and *z* components for the closed helical trajectory of the superluminal quantum model of the electron, the rms values of x, y and z, which are the values of Δx , Δy and Δz in the Heisenberg uncertainty relation, are all calculated to be $R_0 = \hbar / 2mc$, where R_0 is the radius of the closed helical axis of the superluminal electron model. This in itself is a remarkable result, since this result is only found when the radius of the superluminal quantum's helix is $R_0\sqrt{2}$ as used in equations (14) that the values of Δx , Δy and Δz are exactly R_0 . This value $R_0\sqrt{2}$ is the helical radius required to give the electron model the Dirac electron's magnetic moment of one Bohr Magneton.

Calculation of Δp_x and Δp_y for the electron model requires the result used in calculating the electron spin. In that calculation an instantaneous wavelength λ at a point on the closed double-looped helical trajectory was found to be proportional to the radial distance $R(t)$ from the z-axis to that point on the trajectory: $\lambda = \lambda_C \dfrac{R}{R_0}$. In the electron model, the radial value R is give by $R(t) = R_0(1+\sqrt{2}\cos\omega_0 t)$. So

$$\lambda = \lambda_C \frac{R}{R_0} = \lambda_C \frac{R_0(1+\sqrt{2}\cos\omega_0 t)}{R_0} = \lambda_C(1+\sqrt{2}\cos\omega_0 t) \tag{24}$$

The component of the instantaneous momentum *p* in the x-y plane was postulated to be $p = h/\lambda$. So

$$p = h/\lambda = \frac{h}{\lambda_C(1+\sqrt{2}\cos\omega_0 t)} = \frac{mc}{1+\sqrt{2}\cos\omega_0 t} \tag{25}$$

The x and y components of p are then given by

$$\begin{aligned} p_x(t) &= -mc\frac{\sin 2\omega_0 t}{1+\sqrt{2}\cos\omega_0 t} \\ p_y(t) &= mc\frac{\cos 2\omega_0 t}{1+\sqrt{2}\cos\omega_0 t} \end{aligned} \tag{26}$$

Testing the superluminal electron and photon models

Testing for the existence of an electron's superluminal quantum with its *Zitterbewegung* angular frequency $\omega_{zitt} = 2mc^2/\hbar = 1.6\times10^{21}/\sec$ could require great ingenuity. Yet this *Zitterbewegung* angular frequency comes directly from the solution to the relativistic Dirac equation for a free electron.

It might be objected that since in the present models for the electron and the photon, the proposed quanta always travel superluminally, these models violate the theory of relativity's upper limit of *c* for the velocity of the transport of information. But the elementary particles that superluminal quanta form do not themselves travel faster than *c*, so this is not a valid objection to the possible existence of superluminal quanta that form photons or electrons.

Since superluminal quanta would form photons that can have angular frequencies much lower than $\omega_{zitt} = 1.6\times10^{21}/\sec$, one possible approach to testing the superluminal quantum hypothesis is to try to detect them in lower frequency photons, possibly in the frequency range of visible light or microwave radiation. Laser or maser radiation is highly coherent and this could facilitate the localization of helically moving superluminal quanta in individual photons that are all moving in phase. Such measurements are discussed in [13] which concerns detecting *Zitterbewegung* in photons.

There is also a possible test of the superluminal quantum model for the electron. According to this model, an electron and a positron differ in the direction of their internal helicities. If the electron were structured like a circulating left-handed photon, then a positron would be structured like a circulating right-handed photon, and vice versa. Electrons should therefore differentially absorb, scatter or otherwise interact with incoming left and right-handed photons having energies corresponding to the rest mass of electrons.

Conclusions

The photon and the electron are modeled as helically circulating superluminal point-like quanta having both particle-like (E and $\vec{P}$) and wave-like (ω and λ) characteristics. The number of quantitative and qualitative similarities between the Dirac electron with *Zitterbewegung* and the proposed superluminal quantum model of the electron is remarkable, given the relatively simple mathematical form of the superluminal quantum's trajectory. This suggests that the superluminal quantum concept may provide useful physical models for the electron and the photon, and perhaps for other elementary particles as well.

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